# Comments on Fermilab preprint FN-0718

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In a recent publication: "The beam damping dynamics in the optical stochastic cooling", S.Y.Lee, Yunkai Zhang, and K.Y. Ng (LZN) found that optical amplifier with 200 kW of average power must be used for cooling of 36 bunches of protons with $1 \times 10^{11}$ particles per bunch in the TEVATRON at 1 TeV beam energy with damping time of 1400 seconds. Although we recognize that at present time it seems somewhat academic to debate an application of optical stochastic cooling to the TEVATRON, it strike us that their result appears to be different from our earlier assessment of a potential of optical stochastic cooling for the TEVATRON by a large factor approximately $10^5$ .

Attempting to resolve the difference we found the following.

1. LZN used an undulator with 1 T peak magnetic field while 10 T field is technically feasible. This can explain factor of 35.

2. They did not use the optimal focusing of the amplified light in the second undulator and because of that they overestimated the amplifier power by a factor of 16.

3. They used not optimal scenario for damping dynamics and therefore they obtained not optimal particle delays in the bypass lattice. This may explain a factor of 16.

4. While scaling down from a short damping time and high amplifier power case to a long damping time and low amplifier power case they have to consider additional factor of 4.

Taking all factors together and a small factor of 1.3 that accounts for a correction of a numerical coefficient in a formulae for a spontaneous emission in the undulator, we scaled down 200 kW to a little more than 4 Watts. This is reasonably close to what we estimated in the publication [1].

Now let us provide a bit more detail.

1. LZN used in their numerical example an undulator with 1 T peak field. It seems to be a rather conservative approach. The undulator with 10 T peak field and 1.7 m period seems not unreasonable. For example the wiggler magnet (frequency upshifter) with 10 T peak field [2] is scheduled for installation in SPring-8 by Budker Institute of Nuclear Physics in August 2002. Since amplifier power scales as $K^2 / (2 + K^2)$, where K is proportional to the peak magnetic field, using 10 T would relax power requirement by a factor of 35.

It is known that the undulator with a high peak field is at a premium for cooling of heavy particles and it would seem rather natural for us to target the best possible device.

2. It is know from FEL studies that focusing of the laser light in the undulator with Rayleigh length equaled to 1/4 of the undulator length gives the most effective interaction of the particle with light in the undulator [3].

3. LZN analysis of the damping dynamics is not optimal for the following reason.

Recall that the transit time method of optical stochastic cooling uses two undulators, optical amplifier and bypass. In the first undulator each particle radiate the electromagnetic wave.
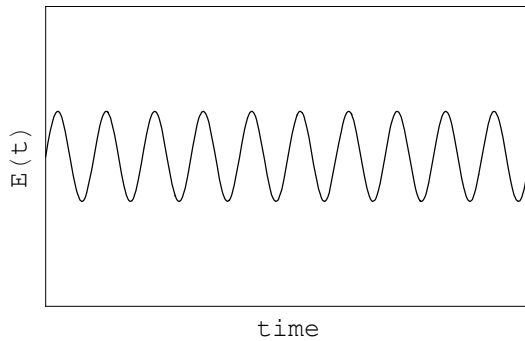


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Note, that all particles radiate the same electromagnetic waves in the first undulator no matter what, "hot" or "cold" beam. Then particles go to the bypass. The bypass provides needed delays for particles to appear in front of the waves of their amplified radiation in the second undulator. The method assumes that there are small variations in these delays from a particle to a particle that are proportional to particle deviations from the equilibrium in energy and betatron x and x' coordinates. These variations play a key role in the cooling. They allow to launch each particle in the right phase of the wave of its own amplified radiation to receive a correcting energy kick proportional to a particle deviation from the equilibrium. For example the equilibrium particle should not receive any kick and therefore its right phase is at the cross of the wave. It turns out that the right phase for a particle with, say, one sigma energy off-set is one radian [1]. (For simplicity we limit this discussion to the case of the energy cooling. An extension to the case of transverse cooling is straightforward ). Positions of two these particles on the wave are illustrated on the plot with red dots.
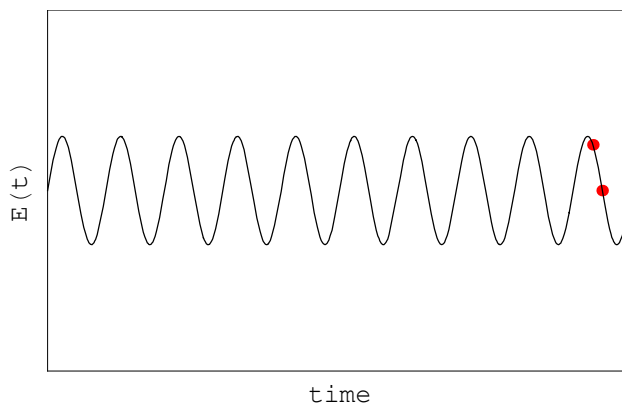


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It is easy to see that if a delay for a particle with one sigma energy off-set is larger than one radian then too many particle will go over the maximum of a sine function and will get smaller than needed energy kicks. Alternatively, if a delay is smaller than one radian than too many particles will stay near the cross of sine function and will also get smaller energy kicks than it is possible.

Now it is easy to explain what LZN had overlooked in what was proposed in [1]. While cooling proceeds, the energy spread of the beam decreases. Particles begin to get lesser and lesser energy kicks since they come closer to the cross of the wave. This is why we proposed to do a continuous adjustment of time-off-flight parameters of the bypass lattice during the damping. By doing this we wanted to keep a particle with one sigma energy off-set at one radian phase position independent of the current beam energy spread. This is essentially the "*damping dynamic*" of optical stochastic cooling. Since an optical wavelength is ~ 1 micron, the adjustment would practically mean less than ~ 0.2 micron change in the pathlength. While energy spread decreases, the amplitude gain of the optical amplifier should also decrease proportionally. The adjustments are not needed at the equilibrium when damping is balanced by some diffusion, like intrabeam scattering or some other type.

LZN treated it differently and according to our estimation they overestimated the optical power by a factor of 16.

We also found that they did not use the optimum in the case of one dimensional cooling. Using achromat lattice in the bypass would eliminate the dependence of particle delays from betatron coordinates. Then their integrals $I_1$, $I_2$ would be zero by definition.

4. When optical amplifier can not support the maximum optimal cooling rate because of a lack of the power, then cooling goes slowly. Inverse cooling time scales as a square root of the amplifier power. This scaling law was correctly used by LZN. But they should also consider the fact that when damping time is much smaller than the optimal damping (more than a factor of 10 in their example), then heating due to other sample particles is negligible and actual cooling goes twice as faster. Therefore for a given damping time their amplifier power requirement must be relaxed by a factor of 4.

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Over the time since publication [1] we made few minor changes in the coefficients in the formulae that were used for assessment of a potential of optical stochastic cooling. They were not essential and we did not consider them worth of a separate publication. However taking together they show a little better result (a factor of $\sqrt{8/e}$ ). This note gives us an opportunity to publish the most current view. In the case when the available amplifier power limits the damping time (the case debated in this note), the number of active passes through the cooling system can be calculated using the following expression:

$$\frac{1}{n^2} = \frac{16}{e} \; \frac{P}{\frac{I}{q}\Delta E} \; \frac{\delta E}{\Delta E} N_u \Gamma$$

Here P is the average amplifier power, I is the average beam current, $e$=2.718, q is the particle charge, $\Delta E = \sigma_e E_b$ , where $E_b$ is the beam energy and $\sigma_e$ is the rms relative energy spread, $N_u$ is the number of undulator periods, $\Gamma = \Delta\omega/\omega$ is the relative bandwidth of the amplifier ( $N_u\Gamma \leq 1$ is assumed), and $\delta E$ is the energy radiated by the particle in the undulator into the fundamental mode:

$$\delta E = 4.12 \; q^2 \, k \; \frac{K^2}{2+K^2} \left[ J_0\left(\tfrac{1}{2} \tfrac{K^2}{2+K^2}\right) - J_1\left(\tfrac{1}{2} \tfrac{K^2}{2+K^2}\right) \right]^2 ,$$

where $J_0$ and $J_1$ are Bessel functions, $K = \frac{qB\lambda_u}{c\,2\,\pi\,Mc^2}$ is the undulator parameter, B is the peak magnetic field, $\lambda_u$ is the undulator period, M is the particle mass, $k = 2\pi/\lambda$ is the wave number, $\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$ is the wavelength of the undulator radiation, and $\gamma = E_b / Mc^2$.

Then the damping time is $\tau = nT$, where T is the time between two active passes through the cooling system.

Using TEVATRON parameters $E_b = 900$ GeV, $\sigma_e = 1.3 \times 10^{-4}$ [4], 36 bunches with $1 \times 10^{11}$ protons per bunch , revolution frequency of 47.7 kHz, 14.5 m long undulator with B=10 T and $\lambda$=800 nm, and assuming Ti:Sapphire amplifier with $\Gamma = 1/N_u$ and one cooling system per turn we found damping time of  1200 sec at the amplifier power of 2 W. A feasibility study of an optical amplifier with this level of power had been reported in [5].

## ■ References

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5. A. Zholents and M. Zolotorev, "An amplifier for optical stochastic cooling", IEEE Particle Accel. Conf. , PAC97, p.1804(1997).